Beamforming With Multi cell Coordination

A thesis submitted in partial fulfilment of the requirement for the degree of

Master of Technology

In

Electronics and Communication Engineering

Specialization: Communication&Networks

By

Venkatesh Chebolu

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Rourkela, Odisha, 769008, India,

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Under the Guidance of

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Department of Electronics and Communication Engineering,

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Rourkela, Odisha, 769008, India,

May 2015.

Dedicated to my parents, brothers and my guide

DEPARTMENT OF ELECRTONICS AND COMMUNICATION ENGINEERING, NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA, ODISHA -769008.

CERTIFICATE

This is to certify that the work done in the thesis entitled **Beamforming With Multi cell Coordination** by **Venkatesh Chebolu** is a record of an original research work carried out by him in National Institute of Technology, Rourkela under my supervision and guidance during 2014-2015 in partial fulfilment for the award of the degree in Master of Technology in Electronics and Communication Engineering (Communication & Networks), National Institute of Technology, Rourkela.

Place: NIT Rourkela

Date: 26-05-2015

Dr. Siddharth Deshmukh

Assistant Professor

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DECLARATION

I certify that,

a. The work presented in this thesis is an original content of the research done by myself under the general supervision of my supervisor.

b. The project work or any part of it has not been submitted to any other institute for any degree or diploma.

c. I have followed the guidelines prescribed by the Institute in writing my thesis.

d. I have given due credit to the materials (data, theoretical analysis and text) used by me from other sources by citing them wherever I used them and given their details in the references.

e. I have given due credit to the sources (written material) used by quoting them where I used them and have cited those sources. Also their details are mentioned in the references.

Venkatesh Chebolu

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ABSTRACT

In this work, we study the optimum multi-cell beamforming and we propose an optimized multi-cell downlink beamforming solution in which our objective is to maximize capacity of a cellular network. We first formulate an optimization problem, maximizing the received signal power of every active user in a cell, subjected to limiting the overall interference observed by other users below a specified level. In addition, we also put constraint on maximum transmit power of the serving base station. Next, we need robust downlink design against the imperfect channel state information. So in order to compute robust beamforming vector we accommodate channel estimation error in our formulation. To model the uncertainity between the true and estimated channel coefficients, we consider channel imperfection as error between true and estimated channel coefficients and we assume error is bounded with in an ellipsoidal set.

The resulting formulation is a non-convex optimization problem. Since it is very difficult to solve a non-convex optimization problem we tried to convert non convex problem in to convex optimization problem. So to get a tractable solution for resulting non convex optimization problem, we exploit linear matrix inequality based S-procedure. The final reformulation is solved by using semi definite relaxation. The efficacy of proposed solution in improving cellular capacity and efficient power transmission is shown by simulations.

Contents

ABBREVIATIONS

BS : Base station

LP : Linear program

QP : Quadratic program

QCQP : Quadratically constrained quadratic program

SOCP : Second-order cone programming

SDP : Semidefinite program

CoMP : co-ordinated multipoint transmission

CSI : Channel State Information

CSIT : CSI at the transmitter

INF : infimum

LMI : Linear Matrix Inequalities

NOMENCLATURE

S^m : Positive semi definite cone

$\mathrm{R}^{m\times n}$: Set of m×n matrices

Y_{opt} : set of optimal values

$\mathbb{R}^n$: sets of n dimensional real vectors

$\mathbb{C}^n$: sets of n dimensional complex vectors

$\mathbb{H}^n$: sets of n dimensional complex Hermitian matrices

W : Used to Describe a matrix

w : Used to Describe a vector

$(.)^H$: Conjugate transpose

$(.)^T$: The transpose

Tr(**A**) : Trace of matrix A

E(.) : Expectation operation

LIST OF FIGURES

Chapter-1

1. Introduction

1.1. Concept of Beamforming

Beamforming is a powerful and efficient technique to transmit and receive the desired signal in presence of co-channel interference by exploiting spatial diversity [11]. It is a signal processing technique in which maximum amount of energy will be transmitted in the desired direction. We have different types of beamforming namely transmit beamforming, receive beamforming, network beam forming etc. Downlink beam forming is one of the best technologies for present and future cellular networks. In cellular networks downlink beamforming is used to transmit the data from the base station (BS) to desired user by deploying multiple antennas at the serving base station. Downlink beamforming is gives an effective solution in reducing the multi-access interference. The concept of beamforming can be briefly explained as follows

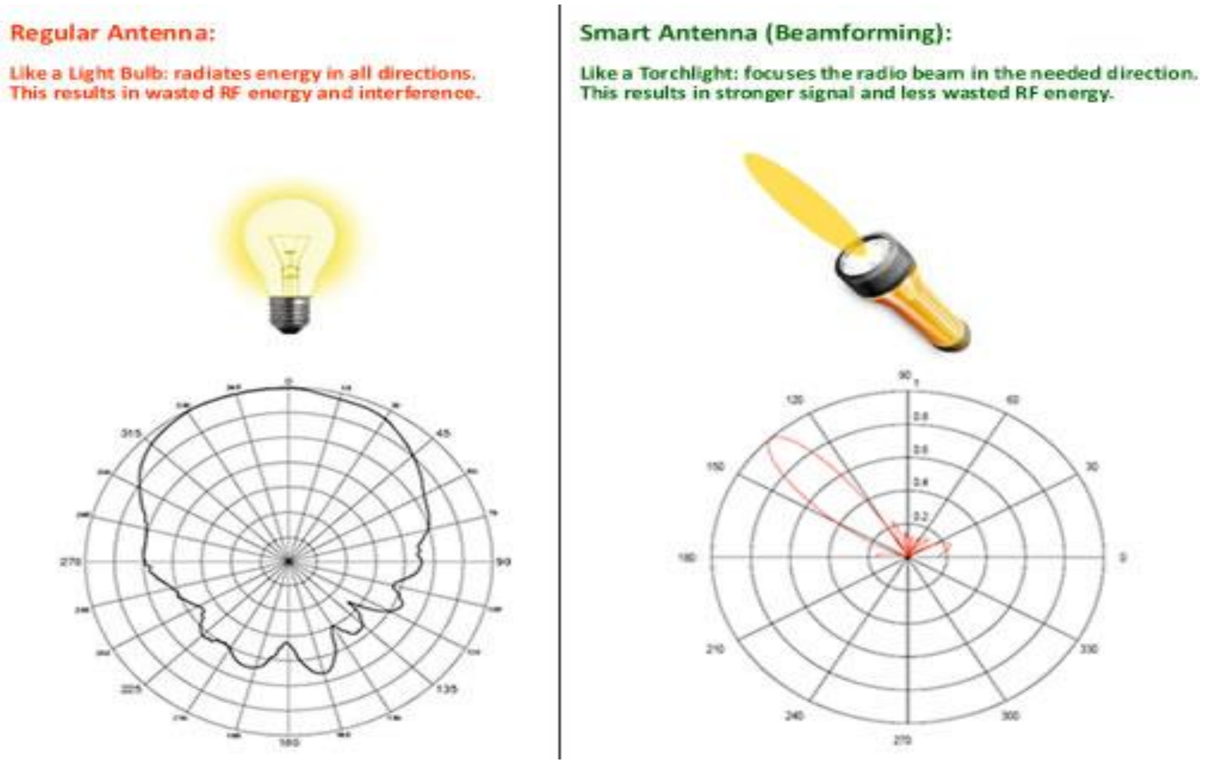


Figure 1-1: Radiation pattern of Normal and Beamforming antenna

Above figure explains how the beamforming transmission will be, if we take a normal antenna, it radiated the whole energy in all directions equally like an electrical bulb transmits the light energy in all directions but the beamforming antenna transmits the energy in the desired direction like a torch light does as shown in the above figure. So in communication systems if we use the beamforming techniques we can have the transmission in the desired direction from the transmitting antenna. Now in beamforming, to achieve the signal transmission in the desired direction we need multiple antennas at the transmission means multiple antennas together gives the energy in the form of beam in the desired direction. This can be explained as follows, so first take an Omni directional antenna at the transmitter and start transmitting it, now we can observe the radiation in all directions from the transmitting antenna as shown in the below figure

1.2. Radiation pattern for single omni directional antenna

Figure 1-2: Radiation pattern for single omni directional antenna

Now when we add the another Omni directional antenna at the transmitter and assume that both antennas are transmitting at the same time, since both antennas transmitting at the same time, the radiation coming from the both antennas interfere each other as shown below. So when the radiations from the two antennas interfere each other, there is a possibility of two types of interference one is constructive interefernce and other is destructive interference. If the phases of two interfered signals

1.3. Radiation pattern for two omni directional antennas

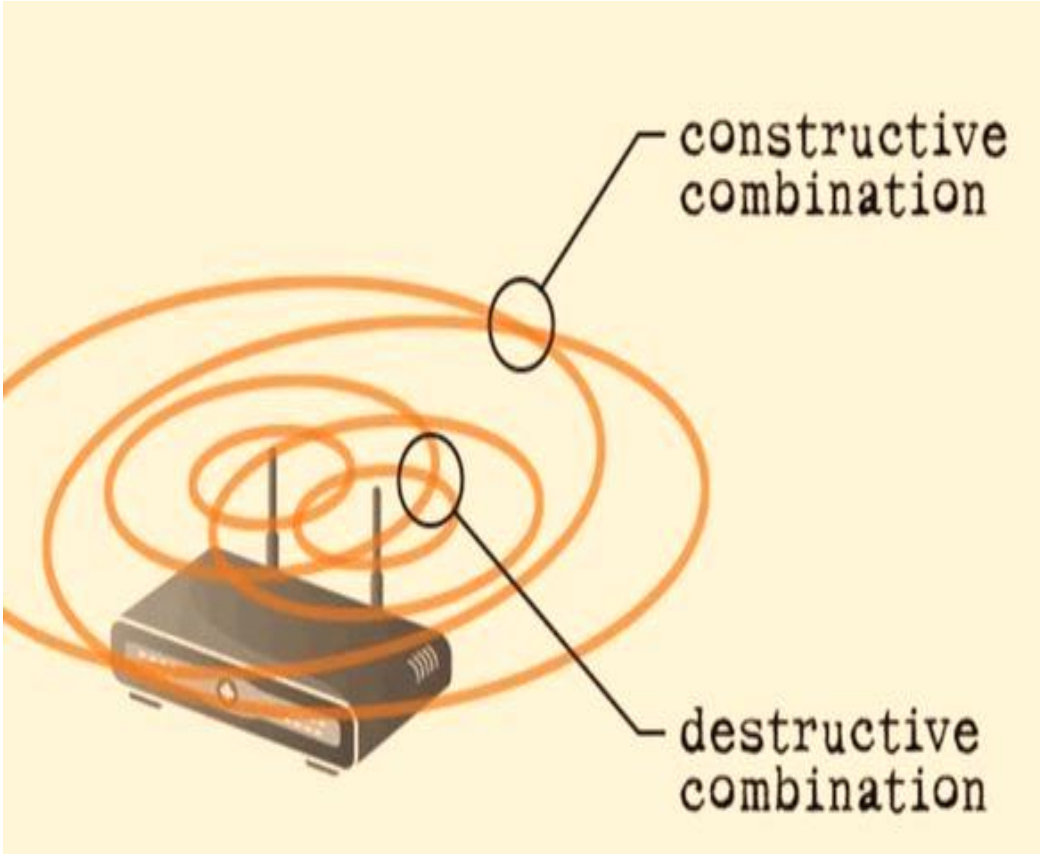


Figure 1-3 : Radiation pattern for two omni directional antennas

are same then the constructive interference takes place then amplitude of the resultant signal is the addition of both the signals. If the two interfered signals are at the out of phase then the destructive interference takes place, in this case the amplitude of the resultant signal is the subtraction of both the signals. This destructive interference is also be useful in certain situations. So if the amplitudes of both the interfered signals are equal then the amplitude of the resultant signal will be zero in the case of destructive interference.

1.4. constructive and destructive combinations

Suppose if we two transmitters which transmitting the sinusoidal waves having same amplitude at the same time, now when constructive interference occurs then the amplitude of the resultant signal will be doubled and when

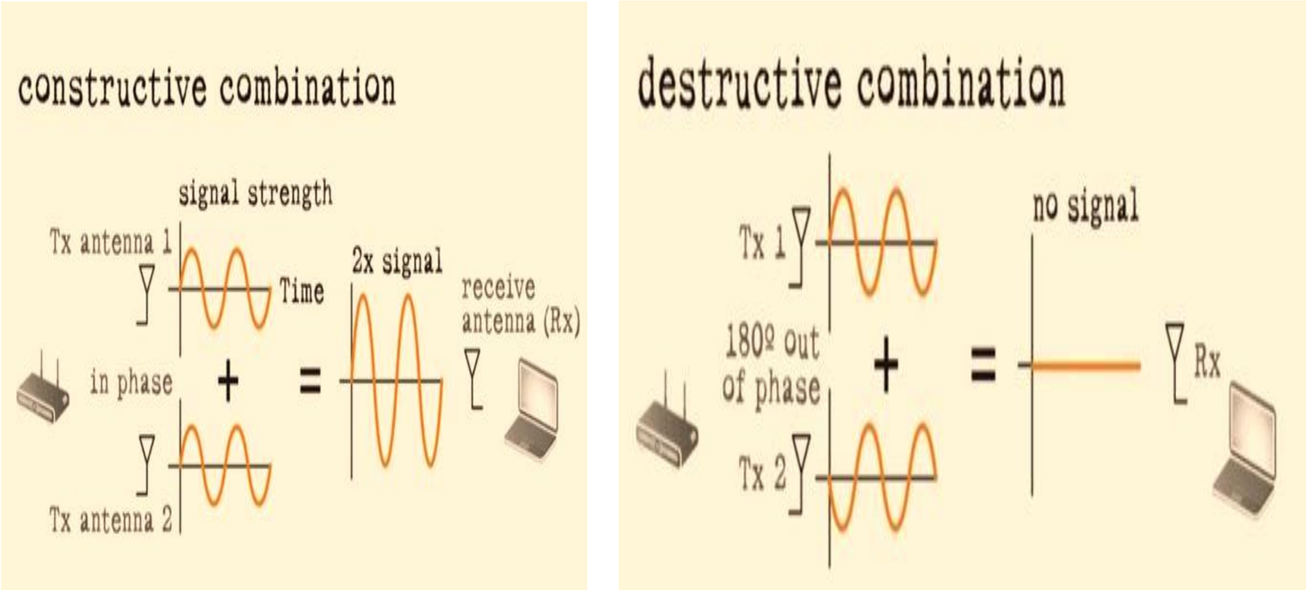


Figure 1-4: constructive and destructive combinations

destructive interference takes place then the amplitude of the resultant signal will be zero as shown in the above figure. This concept will lead us to the concept of beamforming. Because if we have a proper control at all antennas of the transmitter, we can control the phase of the signal radiated by each and every antenna which is very helpful for achieving the beamforming technique in the communications. If we set the phases of the signals radiated by each antenna in such way that constructive interference takes place in the desired direction and the destructive interference takes place in all remaining directions. This technique is called beamforming. In this way by using multiple antennas at the transmitting antenna and control the phases of the signals radiated from each antenna we can achieve beamforming. So we need control over phase and amplitudes of the radiated signals to get the signal in desired direction there by to achieve beamforming. The control over the phases and amplitudes of the radiated signals from each antenna can be achieved by multiplying the transmitted signal with the complex weights and then do the transmission. This should be done at each antenna at the transmitter. This can be explained with the help of below figure.

1.5. Method of Practical generation of beamforming signal

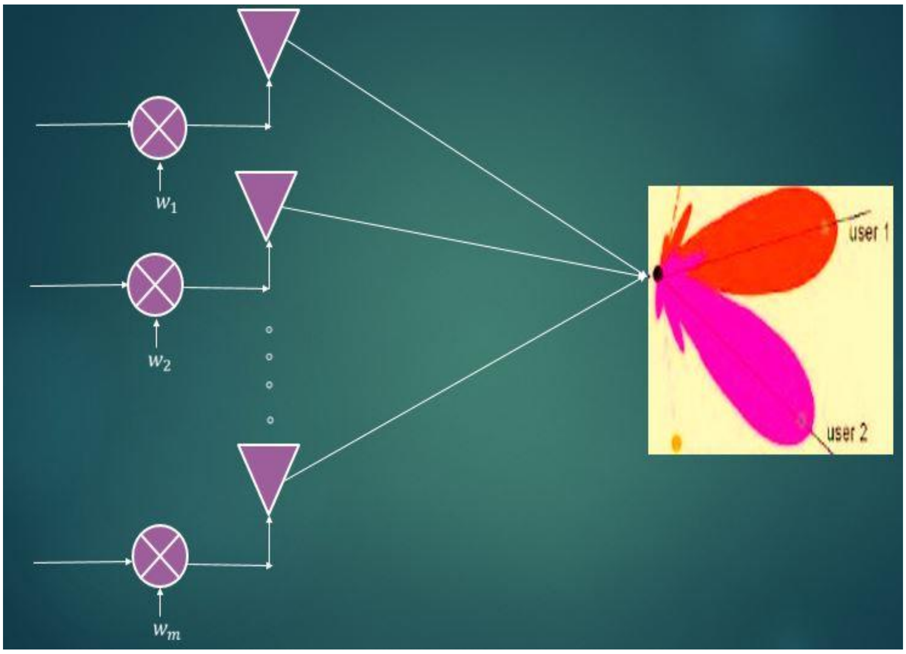


Figure 1-5: Method of Practical generation of beamforming signal

The above figure explains how we can achieve beamforming by multiplying the complex weights with the transmitted signal at the each antenna at the transmitter. Here we are using m number of antennas at the transmitter. Now we each signal is given to the respective antenna but before that each signal is multiplied with some complex weights. So we should choose those complex weights such that well get constructive interference in the desired direction and destructive interference in all other directions. In this way beamforming can be achieved and resulting radiation pattern is as shown in above figure.

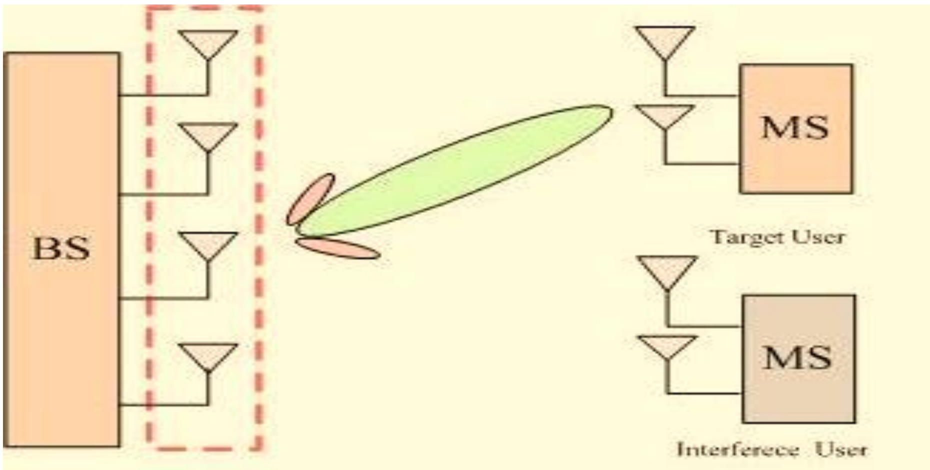


Figure 1-6: Downlink beamforming

1.6. Advantages of beamforming

1) Beamforming technique has many applications in differrent fields like wireless communications, radar, biomedicine, sonat etc…

2) By using beamforming signal to interference noise ratio will be improved

3) signal range can also be improved by using beamforming

4) By using beamforming we can reduce the co channel interfernce and as well as intercell interference

5) Since cochannel and intercell interference are improved, overll network efficiency and capacity of the network will also be improved with the help of beamforming

6) Beamforming enables the high data rates to the user in any communication system

1.7. Thesis organization

Thesis of this work is organized as foloows. In this chapter 2 contains the concepts of convex optimization in which we study the definition of convex set and differrent kinds of convex sets like polyhedra, half space, hyperplane, convex cone etc..after that in this section we also study the definition of convex function and differrent type of convex functions and we will see the differrent types of optimization problems which are very useful for solving oiptimization problems. In chapter 3 we study the concepts of coordinated multi point transmission, centrlized and de centralized CoMP. Here we also study the concept of robust downlink beamforming. After that we formulate a problem for capacity maximization in a cellular network. In chapter 4 first we will do the simulation setup and solve the problem formulated in chapter 3 using convex optimization solving techniques. Now by using this solution we will plot the results shown in chapter 4. Conclusion and future work is discussed in chapter 5.

1.8. Literature survey

Recently many robust methodologies against channel imperfections have been developed for cellular networks. For example, authors in [17] have proposed multicellular downlink beam former design with the objective of minimizing the sum power used by each BS to transmit data to local users. In addition CSI imperfection is considered and signal to interference plus noise ratio (SINR) is maintained above the desired level. Solution for the above problem has been obtained with the help of S-procedure and semi definite relaxation (SDR). In [18] authors studied multi-cell wireless networks and especially focussed on joint robust transmission optimization over the cells. They aimed at maximizing the minimum worst case rate of the network with imperfect CSI. Optimized solution has been achieved by converting the problem in to convex problem and solving it by employing convex optimization problem solvers. In [19] authors minimize the total transmit power at BSs by taking equivocation rates and individual SINR as constraints. In [20] authors minimize the total downlink power at BSs by restricting the SINR outage probability below a threshold.

Chapter-2

2. Concepts of Convex optimization

2.1. Introduction

2.1.1. Mathematical optimization

General form of mathematical optimization can be written as

minimize $g_0(x)$

Subject to $g_j(x) \leq d_i$, j=1, 2,…..n.

Where $g_0(x)$ can be called as objective function

x=$x_1, x_2, \ldots\ldots, x_n$ are the optimization variables or decision variables. Without loss of generality d_i could be zero. We can construct d_i from $g_j(x)$ will give new g_j.

$g_0: R^n \to R$ is objective function g_j

$g_j: R^n \to R$, j=1,2,…..n are the constant functions

So if we solve above optimization problem we will get solution in the form of **x** is smallest value of g_0 among all functions which satisfy the constraints and we can call the solution x as the optimal solution which satisfies both constraints and the objective. We can have a problem that we may have no solution or single solution or multiple solutions.

Solving optimization problems

Solving optimization is very difficult to solve. So to solve above optimization problems, means methods to solve optimization problems involve two categories of compromises. one is very long computation time second one is not always finding the solution. But we have hige exceptions to solve optimization problems. There are some cases where we can solve these problems. Those are

1) Least square problems

2) Linear programming problems

3) Convex optimization problems

2.1.2. Least square problems

The general form of linear squares problems looks like

$$\text{minimize} \quad ||Cx\text{-}d||_2^2$$

The above problem is minimizing Euclidian norm square of Cx-d. And we have to choose x to minimize $||Cx\text{-}d||_2^2$. To solve linear squares problems we have super quality, open source, public domain soft wares that will actually carry out to give the solution of least squares problems very easily.

2.1.3. Linear programming

Linear programming problem is minimizing linear function subject to bunch of linear inequalities. The general form of linear programming problem looks like

$$\text{minimize} \quad a^T x$$

$$\text{subject to} \quad b_j^T x \leq c_j,\ j=1,2,\ldots n$$

For solving linear programing problems we don't have analytical formula. But like least squares, for linear programming also we have super quality, open source, public domain soft wares that will actually carry out to give the solution of linear programme problems very easily. Linear programming problems looks like it would be easier to recognise but not.

2.1.4. Convex optimization problem

The general form of convex optimization problem looks like

$$\text{minimize} \quad g_0(x)$$

$$\text{subject to} \quad g_j(x) \leq d_i,\ \ j=1, 2,\ldots..n$$

Here in the convex optimization problem we will minimize objective function $g_0(x)$ subject to some constraints like $g_j(x) \leq d_i$. Here we should note that the functions $g_0(x)$ and $g_j(x)$ have to be convex functions. Convex means if we draw a graph of any function, it should have positive curvature which means graph should bend upwards. Least square problem has that form because if we plot the least squared objective, basically its looks like a bowl and if we take a slice at levels we get an ellipsoid so it is convex. Similarly linear programming is also convex problem because all the objectives are linear. Linear functions are convex. If we want to solve a convex optimization problem there are no analytical solution. In this also we have super quality, open source, public domain soft wares that will actually carry out to give the solution of convex optimization problems very easily. We have many tricks to convert non convex problems in to convex problems.

2.2. Convex Sets

2.2.1. Affine set

Affine set can be defined as a set where any two distinct points and line through them are all inside the set. We can parameterise the line going through those two points using the parameter such as $\emptyset$. $\emptyset$ in the below parametric form can be varied whole real line. Parametric form for the affine set can be written as

$$x= \emptyset x_1 + (1-\emptyset)x_2 \quad (\emptyset \in R) \tag{2.1}$$

2.2.2. Convex set

It can be defined as a set where any two distinct points and the line segment through them are all inside the set. We can parameterise the line segment through those two points using parameter such as$\emptyset$. Here $\emptyset$ will not have whole real values, but it will only have the values between 0 and 1. Parametric form for the convex set can be written as

$$x= \emptyset x_1 + (1-\emptyset)x_2 \quad \text{with } 0\leq \emptyset \leq 1 \tag{2.2}$$

Examples of convex set

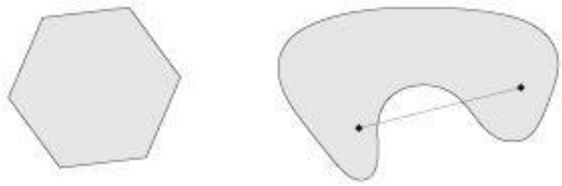


FIGURE 2-1: Examples of Convex Set

2.2.3. Convex combination and convex hull

If $y_1, y_2 . . . . . . y_k$ are the elements of some set A, then convex combination of these elements can be defined as

$$y = \emptyset_1 y_1 + \emptyset_2 y_2 + \ldots\ldots\ldots\ldots \emptyset_k y_k \text{ with } \emptyset_1 + \emptyset_2 + \ldots\ldots\ldots + \emptyset_k = 1 \qquad (2.3)$$

set of all possible convex combinations of the points in the given set A can be called as convex hull of the set A. Figures below shows the examples of convex hulls for two different sets.

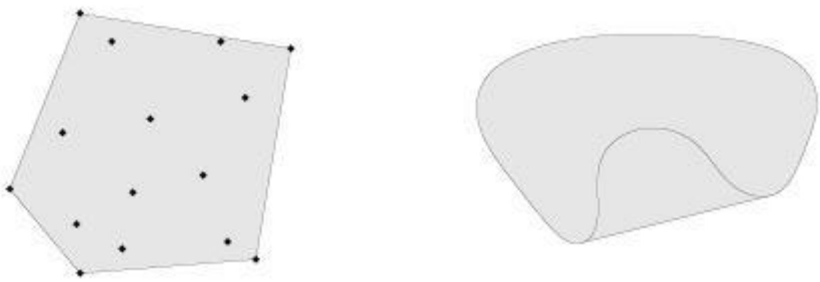


FIGURE 2-2: Convex Hull

2.2.4. Convex cone

If y_1 *and* y_2 are the elements of some set A, then conic combination of these elements can be defined as

$$y = \emptyset_1 y_1 + \emptyset_2 y_2 \quad \text{ with } \emptyset_1, \emptyset_2 \geq 0 \qquad (2.4)$$

It is a special division of linear combination of points. The below figure is a convex cone represents the set of points which satisfies above equation for different combinations of $\emptyset_1, \emptyset_2$. Convex cone is a convex set.

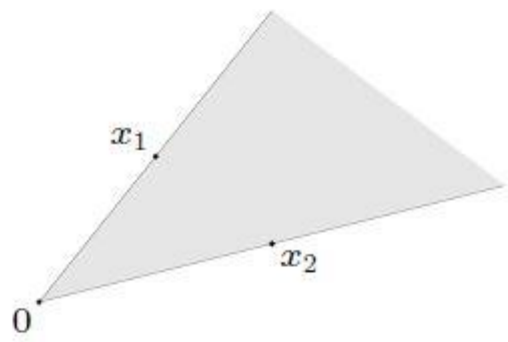


FIGURE 2-3 : Convex Cone

2.2.5. Hyper planes and Half spaces

Hyper plane is a set having the the form { y | $c^T x = d$}, Where d is a constant. And any point which satisfies equation $c^T x = d$ will also lie in hyper plane.

Half space is a set having the form { y | $c^T x \leq d$ }, Where d is a constant and c is a normal vector should not be zero. It is similar to hyper plane but we just take area on one side of that plane. Half space is not wicked space. It is not subspace. Hyper planes are affine and convex whereas half spaces are convex but not affine.

2.2.6. Euclidian balls and Ellipsoids

Euclidian ball with centre x_c and radius a can be defined as

$$S\,(x_c, b) = \{\ y\ \mid \|\, y - x_c\|_2\ \} = \{\ x_c + bv \mid\ \|v\,\|_2\} \qquad (2.5)$$

Ellipsoid is set having the form

$$\{\ y \mid (y - x_c\,)^T\, Q^{-1}\,(y - x_c\,) \leq 1\ \} \text{ with } Q \in S^n_{++} \qquad (2.6)$$

In above equation Q is a positive semi definite matrix. Ellipsoid is generalization of Euclidian ball. Ellipsoid and Euclidian balls are convex sets.

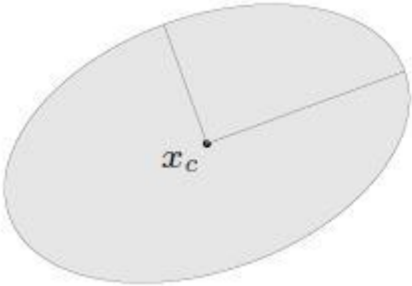


FIGURE 2-4: ELLIPSOID

2.2.7. Norm

Norm is any function denoted by $\| . \|$ that satisfies the following conditions,

1) $\| y \| \geq 0$; $\| y \| = 0$ if and only if y=0

2) $\|m\, y \| = |m|\, |y|$ for m∈R

3) $\|u + v\| \leq \| m \| + \| v \|$

2.2.8. Norm balls and Norm cones

For general norms we can define norm ball. This ball does not look like a ball, it is the ball in the general sense. Norma ball with centre y_c and radius b can be defined as

$$\text{Norm ball:} \quad \{y \mid \| y - y_c \| \leq r\} \tag{2.7}$$

we defined the norm ball very much similar as we define the Euclidian ball but they are going to look different. Norm cone is any point (y,u) where norm of the y is less than or equal to u. This can be written as

$$\text{Norm cone:} \{ (y,u) \mid \| y \| \leq u \} \tag{2.8}$$

2.2.9. Polyhedra

It is a solution set of finitely many linear inequalities and equalities. we can represent it as

$$P: \{y \mid My \leqslant n \, , Ry \leqslant t\} \tag{2.9}$$

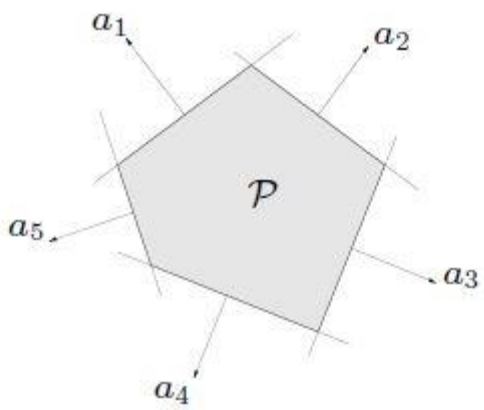


FIGURE 2-5: Polyhedra

The above figure is a example of Polyhedra which is intersection of finite number of half spaces and hyperplanes. In the above figure we got 5 half spaces defined by normal vectors a1 to a5. So if we find the intersection of all half spaces regarding a1 to a5 we will get the shape as shown in the above figure. Polyhedra is a convex set.

2.2.10. Positive semi definite cone

Generally the set of m×m symmetric matrices can be denoted as S^m. And this S^m is a convex set and affine set. Now positive semi definite cone can be defined as

$$S_+^m = \{ Y \in S^m \mid Y \geqslant 0\} \tag{2.10}$$

By seeing above equation we can say that positive semi definite cone is the set of positive semi definite matrices and it is a convex cone.

2.2.11. Proper cone

A Convex cone M ⊆ R^n can be called as a proper cone if it satisfies the following conditions

1) The cone M should be a closed set

2) M should not contain any empty space in the interior

3) M should be pointed means it should not contain any line

Closed set means if we have any shape and the set includes the boarder of that shape with the boundary of that shape with the boundary of that set then it is a closed set. As per the above second above condition interior means inside the set means set not including the boundary.

2.3. Convex functions

2.3.1. Convex function

A function g: R^n →R can be called as a convex function[1], if domain of function g is a convex set and satisfies the following condition

$$g\ (\emptyset u + (1-\emptyset)v) \leq\ \emptyset\ g(u) + (1\text{-}\emptyset g(v)) \qquad \forall\ u,v \in \text{dom } g,\ 0 \leq \emptyset \leq 1 \qquad (2.11)$$

In terms of the graph, let us assume we have a graph of the particular function as shown below, then draw a chord for the graph then if chord lies above the graph then we can say that the function is convex.

Figure2-6: Graph of a convex function

2.3.2. Examples of convex functions

1) Affine function i.e Cx+d on R for any c,d ∈ R is a convex function

2) Exponential function exp(by) for b beloongs to real number set is convex function

3) Powers of the form y^b for b ≥ 1 or b ≤ 0 is always convex function

2.3.3. Examples of convex functions on R^n and $R^{m\times n}$

1) Affine function having the form b^Ty+c ,general form of affine function on R^n is a convex function, where y and c are the vectors and b is a matrix

2) Any norm is a convex function

3) Affine function on $R^{m\times n}$ i.e affine function for matrices is also a convex function

4) Maximum singular norm having the form $\| X \|_2$ is also a convex function.

2.3.4. First order condition for convexity

This first order condition for convexity is very important and it actually a hint as to why convex optimization actuallry works very very well.This says that if any function g is differentiable and has a convex domain then that function will be convex[1], if

$$g(v) \geq g(u)+ \nabla g(v)^T(v-u) \quad \forall\, u,v \in \text{dom } g \tag{2.12}$$

2.3.5. Second order condition for convexity

A function g is twice differentiable if the domain of the function g is open and the hessian of the function must be belongs to S^n. Now this second order condition says that if a function g is twice differentiable, then that function can be convex if hessian of that function is convex i.e

$$\nabla^2 (g(y)) \geqslant 0 \quad \forall\, y \in \text{dom } g \tag{2.13}$$

Quadratic function, quadratic over linear , log-sum-exp functions are some examples of convex functions which satisfies second order conditions.

2.4. Convex optimization problems

2.4.1. Optimization problem in standard form

minimize $g_0(y)$

Subject to $g_j(y) \leq 0$, j=1, 2,…..n

$$p_j(y) = 0,\ \text{j=1, 2,…..m}$$

Where y ∈ R^n is optimization variable

$g_0 : R^n \to R$ is objective function

$g_j: R^n \to R$, j=1,2,…..n are the inequality constraint functions

$p_j: R^n \to R$, j=1,2,…..m are the equality constraint functions

The optimal value for the above optimization problem is

$$a^* = \inf \{g_0(y) \mid g_j(y) \leq 0,\ \text{j=1,2,….n},\ p_j(y) = 0,\ \text{j=1, 2,…..m}\} \tag{2.14}$$

The optimal value a* is infinite if all the constraints are not satisfied by any y in the problem**,** because infimum of empty set is infinite. Similarly a* is negative infinite if the sequence of y which are feasible with $g_0(y_i)$ is going to negative infinite.

2.4.2. Optimal and locally optimal points

A point y can be called as feasible point if it is in the intersection of the domains of both objective and constraints and satisfies the constraints if the optimization problem. Now the point y is optimal point if the value of objective function at y is optimal value of the problem. Y_{opt} is the set which contains the optimal points of the problem. Y_{opt} is empty if the problem is infeasible or unbounded below.

A point y is locally optimal when we add a constraint which contains positive R such that

$$\text{minimize (over v)}\ \ g_0(v)$$

$$\text{Subject to}\quad g_j(v) \leq 0,\ \ \text{j=1, 2,…..n}$$

$$p_j(v) = 0,\ \ \text{j=1, 2,…..m}$$

$$\| v - y \|_2 \leq \text{R}$$

Implicit constraints

First consider normal optimization problem minimize $g_0(y)$

Subject to $g_j(y) \leq 0$, j=1, 2,…..n

$p_j(y) = 0$, j=1, 2,…..m

In the above problem $g_j(y) \leq 0$, $p_j(y) = 0$ are called as explicit constraints. The above problem has not only explicit constraints but also implicit constraints. The implicit constraint in the above problem is explained as follows. Actually in above problem we cannot take every y that is satisfying objective and constraints as an optimal solution, we should take set of y which are from the domain of objective and constraints and from the set we have to check which y is satisfying both constraints and the objective. So the constraint that y should be from the intersection of the domains of both constraints and objective is implicit constraint in the above problem.

2.4.3. Feasibility problem

Consider an optimization problem as shown below

find y

subject to $g_j(y) \leq 0$, j=1, 2,…..n

$p_j(y) = 0$, j=1, 2,…..m

The above problem is special of optimization problem which can be rewritten as

minimize 0

subject to $g_j(y) \leq 0$, j=1, 2,…..n

$p_j(y) = 0$, j=1, 2,…..m

The solution of the above problem is 0 if constraints are feasible other wise solution is infinite if constraints are infeasible.

2.4.4. Convex optimization problem

The standard form of optimization problem can be written as [2]

$$\text{minimize} \quad g_0(y)$$

$$\text{subject to} \quad g_j(y) \leq 0, \quad j=1, 2,\ldots..n$$

$$c_j^T y = d_j, \quad j=1, 2,\ldots..m$$

The above optimization problem is convex optimization problem if the objective and set of inequality constraint functions must be convex functions and the equality constraints must be affine functions. We

2.4.5. Optimality criterion for differential g_0

In a convex optimization problem if the objective function $g_0(y)$ is differentiable then y is optimal point when y is feasible and

$$\nabla g_0(y)(z-y) \geq 0 \ \ \forall \text{ feasible } z$$

Equality constrained problem

The basic form of the equality constraint problem is as follows,

$$\text{minimize} \quad g_0(y)$$

$$\text{subject to} \quad CY=D$$

It is also a convex optimization problem but without inequality constraints. Now for the above type of equality constrained problems the solution y is optimal when there exists a m such that

$$Y \in \text{dom } g_0 \text{ and } CY=D, \quad \nabla g_0(y)+ C^T m = 0 \qquad (2.15)$$

Equivalent convex problems

Two problems are said to be equivalent if the solution of one problem is obtained with the modest effort from the solution of the other problem and vice versa. We have some transformations those preserve convexity.

a) Eliminating the equality constraints

We can get the equivalent problem of original problem by eliminating the equality constraint from the original problem. Suppose if we have the problem as shown below,

minimize $g_0(y)$

subject to $g_j(y) \leq 0$, j=1, 2,…..n

CY=D , j=1, 2,…..m

Now we can write the equivalent form for the above problem by eliminating the equality constraints. In the above problem we can eliminate CY=D and the way we do is that, we found a matrix G and point y_o so that CY=D is equivalent to y = Gz + y_o. Now we can make a new problem by replacing y = Gz + y_o in the above problem by removing CY=D then we will get the equivalent for the above problem as

minimize (over z) $g_0(\mathrm{Gz} + y_o)$

Subject to $g_j(\mathrm{Gz} + y_o) \leq 0$, j=1, 2,…..n

If the above problem with equality constraint is convex then its equivalent problem with its equality constraint is also convex because the functions in equivalent problem is affine composition of functions in original problem which preserves the convexity. Both original and equivalent problem are only equivalent but not identical.

b) Introducing equality constraints

Now let us do the reverse operation to the above operation. Let us take one optimization problem and add equality constraint, adding equality constraint causes in increasing the variables and solving such type of problems are difficult compared to the problems which do not have equality constrain also very useful in solving optimization problem in many cases. Now let us take the problem as shown below

minimize $g_0(C_o u + d_o)$

Subject to $g_j\left(C_j u + d_j\right) \leq 0$, j=1, 2,…..n

Now the equivalent problem for the above problem by adding equality constraint is

$$\text{minimize(over } u_{i,} v_i) \quad g_0(v_0)$$

$$\text{subject to} \qquad g_j(v_j) \le 0, \qquad j=1, 2, \ldots n$$

$$v_j = C_j\, u + d_j \qquad j=1, 2, \ldots n$$

So we introduce new variable v_0 in place of $C_o u + d_o$ and v_j in place of $C_j\, u + d_j$. And here the problem started with no inequality constraints after the elimination step we have added variables and added equality constraints. This is not look like progress. But this method is first step to lots of progress.

c) Introducing slack variables for linear inequalities

We can write the equivalent form to the optimization problems by introducing the new variables called slack variables. First let us take the normal optimization problem as shown below.

$$\text{minimize} \quad g_0(y)$$

$$\text{subject to,} \quad c_j^T \text{y} \le d_j, \quad j=1, 2, \ldots n$$

Now above problem is equivalent to (after introducing slack variables)

$$\text{minimize (over y and m)} \quad g_0(y)$$

$$\text{subject to,} \qquad c_j^T \text{y} + m_j = d_j, \quad j=1, 2, \ldots n$$

$$m_j \ge 0 \qquad , \quad j=1, 2, \ldots n$$

In the above constraint c_j^T y $\le d_j$ is converted to c_j^T y + $m_j = d_j$ (equality constraint) and m_j is the slack variable where $m_j = d_j$ - c_j^T y and we will write slack variables bigger than equal to 0 or to put in to our

2.4.6. Linear program (LP)

The standard form of the linear program optimization[3] problem is

$$\text{minimize} \quad e^T \text{y} + \text{f}$$

$$\text{subject to} \quad \text{Uy} \le \text{v}$$

My=n

Linear program problem is a convex optimization problem in which it has affine function as an objective function and LP has some constraints. The feasible set of linear program problem is a polyhedron.

Example of LP:

Piece wise linear minimization

The basic form of the Piece wise linear minimization can be written as

$$\text{minimize} \quad max_{j=1,2,\ldots\ldots m}\,(C_j\,y + d_j)$$

Now the above problem can be equivalently written as Linear Program problem as

$$\text{minimize} \quad v$$

$$\text{subject to} \quad C_j\,y + d_j \le v \qquad j=1, 2,\ldots..m$$

2.4.7. Linear fractional program

The generalized linear program looks like this

$$\text{minimize} \quad g_0(y)$$

$$\text{subject to} \quad Uy \le v$$

$$My=n$$

Now linear fractional program is

$$g_0(y) = \frac{p^T y + q}{r^T y + t"} \qquad \text{dom } g_0(y) = \{y \mid r^T y + t > 0\} \qquad (2.16)$$

Linear fractional programme is generally a quasi-convex optimization problem and the above problem can be solved by using the method of bisection. Now we can write an equivalent linear program problem form for the above linear fractional program as

$$\text{minimize} \quad p^T a + qb$$

$$\text{subject to} \quad Ua \leqslant vb$$

$$Ea = Fb$$

$$r^T a + tb = 1$$

$$b \geqslant 0$$

2.4.8. Quadratic program(QP)

The basic form of the quadratic program problem[4] is

$$\text{minimize} \quad \frac{1}{2}\, y^T E\, y + f^T\, y + g$$

$$\text{subject to} \quad Uy \leq v$$

$$My = n$$

In above problem E is positive semi definite matrix so objective function in above problem is convex quadratic function. And constraints in above problem forms a polyhedron. So quadratic problem can be described as minimize a convex quadratic function over a polyhedron. If E is zero in above equation we can recover linear program from the above problem so QP is strict extension of LP

Examples of QP:

1) Least squares problem is the best example of quadratic program optimization problem

2) Linear program problem by considering random cost is also the example of QP

2.4.9. Quadratically constrained quadratic program (QCQP)

The basic form of the QCQP is as follows

$$\text{minimize} \quad \frac{1}{2}\, y^T E_0\, y + f_0^T\, y + g_0$$

$$\text{subject to} \quad \frac{1}{2}\, y^T E_j\, y + f_j^T\, y + g_j \leq 0, \quad j=1,2,\ldots\ldots.n$$

$$My=n$$

In the above problem both objective and first constraint is quadratic functions so it can be called as quadratically constrained quadratic problem. Here E_0 and E_j and is a positive semi definite matrices.

2.4.10. Second-order cone programming(SOCP)

The basic form of the second order cone programming [5] is

$$\text{minimize} \quad g^T y$$

$$\text{subject to} \quad \| C_j y + d_j \|_2 \leq A_j^T y + b_j \quad j=1,2,\ldots\ldots.n$$

$$My = f$$

$$\text{Where } C_j \in R^{m_j \times m} \text{ and } M \in R^{v \times m}$$

In the above problem inequalities can be called as second order conic constraints. In the above problem if we take m_j as zero then above SOCP will be transformed to LP problem and if we take A_j=0 then above problem will be transformed to QCQP. We can treat SOCP as more general form compared to LP and QCQP.

2.4.11. Geometric programming

Monomial function: The function g having the form g(y) = a $y_1^{c_1}\, y_2^{c_2} \ldots\ldots\ldots y_m^{c_m}$ where domain of g= R_{++}^n and a>0 and exponent $c_m \in$ R can be called as monomial function.

Posynomial function: posynomial function can be defined as the sum of the monomials which can be defined as

$$g(y)=\sum_{k=1}^{K} a\, y_1^{c_{1k}}\, y_2^{c_{2k}} \ldots\ldots\ldots y_m^{c_{mk}} \qquad \text{dom } g= R_{++}^n \tag{2.17}$$

Now the basic form of the geometric programming is minimize $g_0(y)$

Subject to $g_j(y) \leq 1$, j=1, 2,…..n

$p_j(y) = 1$, j=1, 2,…..m

Where the function $g_j(y)$ should be posynomial and $p_j(y)$ should be monomial function

2.4.12. Geometric program in convex form

We can convert geometric program problem in to convex optimization problem by doing some simple transformations. For that first of all take $x_j = \log y_j$ then take the logarithmic cost,

Now the monomial constraint g(y) = a $y_1^{c_1}\, y_2^{c_2} \ldots\ldots\ldots y_m^{c_m}$ will be converted to

$$\log g(\, e^{x_1}, \ldots\ldots\ldots\ldots e^{x_n}) = e^T x + f \tag{2.18}$$

and the posynomial constraint g(y)=$\sum_{k=1}^{K}$ a $y_1^{c_{1k}}\, y_2^{c_{2k}} \ldots\ldots\ldots y_m^{c_{mk}}$ will be converted to

$$\log g(\, e^{x_1 k}, \ldots\ldots\ldots\ldots e^{x_{nk}}) = \log(\textstyle\sum_{k=1}^{K} e^{c_k^t x + f_k}) \text{ where } f_k = \log a_k \tag{2.19}$$

Now geometric program will be transformed to convex problem as

$$\text{minimize} \quad \log\,(\textstyle\sum_{k=1}^{K} \exp(c_{0k}^T x + f_{0k}))$$

$$\text{subject to} \quad \log\,(\textstyle\sum_{k=1}^{K} \exp(c_{jk}^T x + f_{jk})) \leq 0 \quad j=1,2,\ldots.n$$

$$Ux+v=0$$

2.4.13. Generalized inequality constraints

We can have convex optimization problem having generalized inequalities instead of normal inequalities which can be shown as follows

minimize $g_0(y)$

subject to $g_j(y) \preccurlyeq_{k_j} 0$, j=1, 2,…..n

Cy=d, j=1, 2,…..m

In the above convex optimization problem g_0: $R^n \rightarrow R$ is a convex function, g_0: $R^n \rightarrow R^{k_j}$ and k_j is the convex with respect to proper cone k_j

Conic form problem:

The standard form of the conic form problem is as follows

minimize $e^T y$

subject to Gy+f $\preccurlyeq_k$ 0

Cy=d

The above is also a special case of optimization problem having objective and constraints as an affine function and constraints are the generalized inequality constraints.

2.5. Semidefinite program(SDP)

The standard form of the semi definite program [6]is as follows

minimize $e^T y$

subject to $y_1 G_1 + y_2 G_2 + \ldots\ldots\ldots + y_n G_n$+F$\preccurlyeq$0

Cy=d

Where G_i , F ∈ S^k

The inequality constraints present in above problem can be called as linear matrix inequalities. Semidefinite program problems may also contain multiple linear matrix inequalities as shown below.

$$y_1 G_1 + y_2 G_2 + \ldots\ldots\ldots + y_n G_n + F \preccurlyeq 0 \qquad (2.20)$$

$$y_1 G_{11} + y_2 G_{22} + \ldots\ldots\ldots + y_n G_{nn} + F1 \preccurlyeq 0 \qquad (2.21)$$

We can write above two linear matrix inequalities as single linear matrix inequality as shown below

$$y_1\begin{bmatrix}G_1 & 0\\ 0 & G_{11}\end{bmatrix} + y_2\begin{bmatrix}G_2 & 0\\ 0 & G_{22}\end{bmatrix} + \ldots\ldots\ldots\ldots\ldots y_n\begin{bmatrix}G_n & 0\\ 0 & G_{nn}\end{bmatrix} + \begin{bmatrix}F & 0\\ 0 & F1\end{bmatrix} \leqslant 0 \qquad (2.22)$$

LP and equivalent SDP

The standard form of linear program is

minimize $b^T y$

subject to Cy$\leqslant$ a

Now the equivalent semidefinite program form for above LP is as follows

minimize $b^T y$

subject to diag(Cy-a)

2.5.1. SOCP and equivalent SDP

The standard form of SOCP is

minimize g^Ty

subject to $\| \; C_j y \; + \; d_j \; \|_2 \; \leq \; A_j^T$y + b_j j=1,2,……..n

Now the equivalent semidefinite program for above SOCP is

minimize g^Ty

subject to $\begin{bmatrix}(A_j^T\text{y} \; + \; b_j)I & C_j y \; + \; d_j\\ C_j y \; + \; {d_j}^T & A_j^T\text{y} \; + \; b_j\end{bmatrix} \geqslant 0$ j=1,2,……..n

2.5.2. Eigenvalue minimization

minimize λ_{max}(B(y))

where B(y) = $B_0 + y_1 B_1 + \ldots\ldots\ldots y_n B_n$(given $B_i \in S^k$)

Now for the above problem equivalent SDP is

minimize v

subject to B(y)⩽ vI

Here variables y∈ R^n, v ∈ R and follows from λ_{max}(B) ⩽ v ↔ B⩽ vI

Chapter-3

3. Problem formulation and implementation

3.1. co-ordinated multipoint transmission[7] (CoMP):

In downlink beam forming inter cell interference has become a major problem to improve the spectrum efficiency in universal frequency reuse networks. . In addition to co-channel interference mitigation, efficient beam former design can improve downlink system capacity and minimize total transmit power [12]. One way to achieve efficient beam- former design is coordination between neighbouring Base stations (BS) s [13]. This technique is also called co-ordinated multipoint transmission (CoMP). CoMP is two types one is centralized CoMP and other one is decentralized CoMP.

3.2. centralized CoMP

In centralized CoMP The central unit does all signal processing tasks with BSs sharing their data and global CSI [14]. In centralized CoMP[8], all users present in the cellular network sends the data and channel state information(CSI) to its local BS then all BS

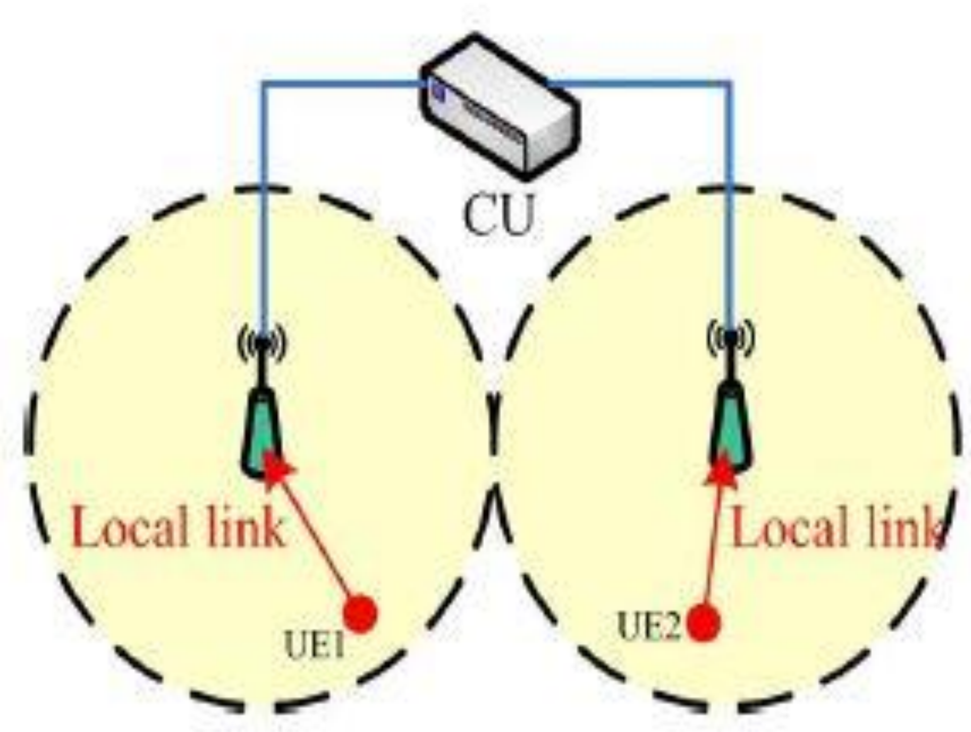


Figure 3-1: Centralized CoMP

sends that data and CSI sent by the respective local users to the central processing unit through ideal backhaul. In this way coordination between the BSs will take place centralized CoMP. But centralized CoMP requires some additional resources of ideal backhaul which is practically impossible. That is the reason why Distributed or decentralized CoMP systems have become a recent research interest as we have practical limitation in centralized CoMP [15].

3.3. De-centralized CoMP

In this frame work each user send its channel state information not only to its local user but also to all BSs present in the cellular network. In this decentralized CoMP [9] we need not to have

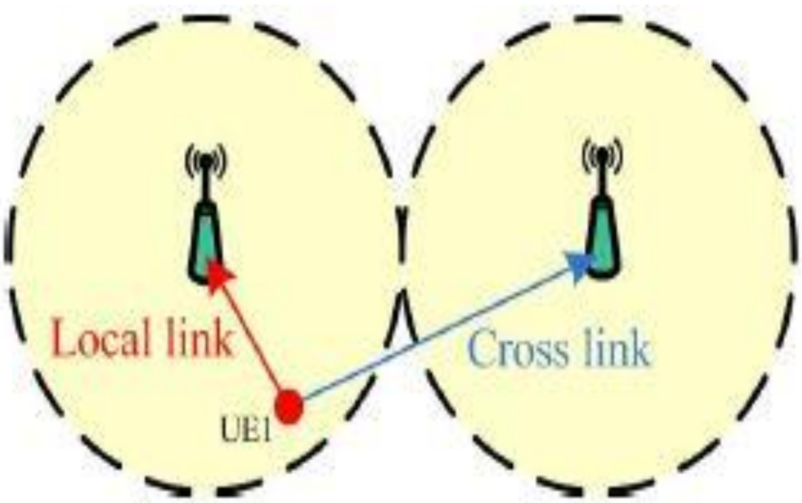


Figure 3-2: Decentralized CoMP

central unit for the coordination between the BSs. And in this method each BS sends the data to its laocal only but it wont send the data to any other BS or central unit. In this we need not to have latency back hauls.

3.4. Robust Downlink beamforming[10]

Acquisition of CSI at the transmitter i.e. BS (CSIT) in downlink beamforming in COMP system is very essential for effective downlink beamforming towards the desired user terminal. But even with sophisticated training, estimated CSI cannot be perfect in the practical systems due to several reasons like estimation error, delay and quantization errors. Hence in addition to design of decentralized CoMP system it is important to find the robust design methodologies against imperfect channel knowledge for accomplishing the effective gains of CoMP in practical scenarios.

Hence in addition to design of decentralized CoMP system it is important to find the robust design methodologies against imperfect channel knowledge for accomplishing the effective gains of CoMP in practical scenarios. In robust formulation of cellular beamforming problem the true CSIT is considered to be confined within an uncertainty region and beamforming vectors are designed such that they remain feasible despite the imperfections in estimated channel [16]. Such problems typically lead to optimization problems with infinite number of constraints and reformulating them to tractable equivalent forms is a challenging task. Recently many robust methodologies against channel imperfections have been developed for cellular networks. For example, authors in [17] have proposed multicellular downlink beam former design with the objective of minimizing the sum power used by each BS to transmit data to local users. In addition CSI imperfection is considered and signal to interference plus noise ratio (SINR) is maintained above the desired level. Solution for the above problem has been obtained with the help of S-procedure and semi definite relaxation (SDR). In [18] authors studied multi-cell wireless networks and especially focussed on joint robust transmission optimization over the cells. They aimed at maximizing the minimum worst case rate of the network with imperfect CSI. Optimized solution has been achieved by converting the problem in to convex problem and solving it by employing convex optimization problem solvers. In [19] authors minimize the total transmit power at BSs by taking equivocation rates and individual SINR as constraints. In [20] authors minimize the total downlink power at BSs by restricting the SINR outage probability below a threshold.

Here in our problem we considers a decentralized CoMP approach in which each BS with in a cell only sends data to its local user. This technique helps in reducing the signalling overhead compared with centralized CoMP. Here we formulate a robust distributed optimization problem that maximizes the received signal power of each user present in a particular cell subject to total transmitted power at the base station remains below the required level and overall interference on the other cell users due to the transmission of BS in given cell should be maintained below a desired level in the presence of imperfect CSI. We considers the imperfections in CSI between true and estimated channel coefficients is confined within a spherical uncertainty set. The worst case solution of the above proposed problem can be obtained by reformulating non convex problem as semi definite relaxation and constraints as linear matrix inequalities.

3.5. System model and problem formulation

We consider a cellular network with N cells. We assume that each cell in cellular network consists of one base station and each base station in the network is equipped with M no of antennas. Further we consider L active single antenna users in each cell of the network. In the cellular network each BS directs the beam to its corresponding users. Objective of the directed beams is to increase the received signal power of desired user and to restrict the inter-cell interference to the other users below a particular value. In addition, it also maintain the total signal power transmitted by the BS below a particular threshold. Let, S_l={ 1,2,……U} is the set of locally active users in particular cell q and S_o={ 1,2,……R} the set of user in adjacent cells which are subjected to inter-cell interference due to the transmission by the q-th BS with in the network. Here we define, $h_i \in \mathbb{C}^{Mx1}$ as the channel vector which consists of channel coefficients between the BS in cell q and active users $i \in S_l$, $g_t \in \mathbb{C}^{M\times 1}$ as channel vector which consists of channel coefficients between the q-th BS and users $t \in S_o$.The received signal at user 'i' is given by,

$$y_i = h_i^H w_i s_i + \sum_{j=1, j\neq i}^{U} h_i^H w_j s_j + v_i + n_i \,, \tag{3.1}$$

where, $w_i \in \mathbb{C}^{Mx1}$is the beamforming vector corresponding to the i-th user and s_i is the complex scalar which denotes the data symbol for user i. The resultant inter-cell interference observed at user 'i' because of the transmission by BSs in the cells other than q is denoted by v_i. n_i denotes the circularly symmetric complex Gaussian random variable at user 'i' which has zero mean and variance σ^2, $n_i \sim \mathbb{CN}(0, \sigma^2)$. We also assume that the average energy of the transmitting symbol s_i is normalized to unity i.e $E_{s_i}(|s_i|^2)$ =1.The expression for the signal to interference plus noise ratio observed by user 'i' can be written as

$$\mathrm{SINR}_i = \frac{|h_i^H w_i|^2}{\sum_{j \neq i} |h_i^H w_i|^2 + \xi_i + \sigma^2}, \tag{3.2}$$

where, ξ_i in above expression is total inter-cell interference experienced by user 'i' due to the BSs in other cells i.e ξ_i=E[$|v_i|^2$]. Next, to maximize the capacity for the desired user, we formulate an optimization problem maximizing the SINR while satisfying the transmit power constraint. Since maximizing SINR is non convex we maximize the received signal power while restricting the undesired interference power to other users. Thus, if every BS restricts interference power to some threshold, the optimization problem to calculate the optimum downlink beamforming vectors at any given BS *q*, can be expressed as

$$\max \quad \left|h_i^H w_i\right|^2$$

$$\text{s.t} \qquad \sum_{t \varepsilon S_o} \sum_{i \varepsilon S_l} \mu_t \; w_i^H \, g_t \, g_t^H \, w_i \;\leq\; K$$

$$\sum_{i \varepsilon S_l} \eta_i \, w_i^H \, w_i \;\leq\; P_q, \tag{3.3}$$

where, K is the maximum allowed interference power at active users $t \in S_o$ due to the transmission of BS *q*. P_q is the maximum allowed transmitted signal power by the BS *q*. The weighting factor needed for the active user $i \in S_l$ is represented by η_i and μ_t required for the adjacent outer-cell users $t \in S_o$ by the BS *q*. These coefficients are used by the scheduler for setting up the priority levels which depends on the quality (cost) of requested services by different users or to proportionally maintain fairness among the users. The objective function in (3) denotes the received signal power at user i, i.e. $i \in S_l$ and left side term of inequality in constraint one in (3) is the overall interference power on the other cell users except users in S_l due to the transmission of BS *q*. Finally left side term of inequality in constraint 2 in (3) is the total signal power transmitted by the BS *q*. In next section, we modify our formulation for robust solution by considering imperfect channel state information.

3.6. Robust downlink beamforming formulation in the presence of CSI

Let us assume, $\hat{g}_t \in \mathbb{C}^{Mx1}$ and $\hat{h}_i \in \mathbb{C}^{Mx1}$ be the estimated CSIs at the BSs. Then the true CSI can be expressed as

$$g_t = \hat{g}_t + e_t, \quad \forall\, t\, \mathcal{E}\, S_o, \qquad (3.4)$$

$$h_i = \hat{h}_i + e_i, \quad \forall\, i\, \mathcal{E}\, S_l,$$

where, $e_t \in \mathbb{C}^{Mx1}$ and $e_i \in \mathbb{C}^{Mx1}$ denotes the CSI error vectors. We considered that the vectors e_t and e_i are bound with in a ellipsoidal sets defined as

$$e_t^H Q_t e_t \leq 1, \quad \forall\, t\, \mathcal{E}\, S_o, \qquad (3.5)$$

$$e_i^H Q_i e_i \leq 1, \quad \forall\, i\, \mathcal{E}\, S_l, \qquad (3.6)$$

where, $Q_t \in \mathbb{C}^{M\times M}$ and $Q_i \in \mathbb{C}^{M\times M}$ are a positive semi definite matrix which characterizes the shape and size of the ellipsoid. Substituting for g_t and h_i from (4) in (3), we will get

$$\max_{w_i} \ \min_{e_i^H Q_i e_i \leq 1} \quad |\left(\hat{h}_i^H + e_i^H\right) w_i|^2$$

$$\text{s.t} \quad \min_{e_i^H Q_i e_i \leq 1} \quad \textstyle\sum_{t \varepsilon S_o} \sum_{i \varepsilon S_l} \mu_t\, w_i^H\, (\hat{g}_t + e_t)(\hat{g}_t^H + e_t^H)\, w_i \leq K$$

$$\textstyle\sum_{i \varepsilon S_l} \eta_i\, w_i^H\, w_i \leq P_q, \quad \forall\, i\, \mathcal{E}\, S_l \qquad (3.7)$$

By introducing slack variable z, we can write (7) as

$$\max \quad z \tag{3.8}$$

$$\text{s.t} \quad |(\hat{h}_i^H + e_i^H)\, w_i|^2 \geq z,$$

$$e_i^H Q_i e_i \leq 1, \;\; \forall \;\; i \,\mathcal{E}\, S_l,$$

$$\min_{e_i^H Q_i e_i \;\leq\; 1} \quad \sum_{t \mathcal{E}\, S_o} \sum_{i \mathcal{E} S_l} \mu_t \;\; w_i^H (\hat{g}_t + \; e_t)(\,\hat{g}_t^H \; + e_t^H)\, w_i \leq K,$$

$$e_t^H Q_t e_t \leq 1, \;\; \forall \;\; t \,\mathcal{E}\, S_o,$$

$$\sum_{i \mathcal{E} S_l} \;\; \eta_i \, w_i^H \, w_i \leq P_q \;.$$

The problem in (8) and its constraints can be rewritten as

$$\min \quad -z \tag{3.9}$$

$$\text{s.t} \;\; \left(\hat{h}_i^H + e_i^H\right) \; \mathbf{W}_i \;\; (\hat{h}_i + e_i) - z \geq 0,$$

$$e_i^H Q_i e_i \leq 1, \qquad \forall \; i \in S_l,$$

$$- \sum_{t \mathcal{E}\, S_o} \sum_{i \mathcal{E} S_l} \mu_t \;\; (\,\hat{g}_t^H \; + e_t^H)\, \mathbf{W}_i (\hat{g}_t + e_t) + K \;\; \geq 0,$$

$$e_t^H Q_t e_t \leq 1, \qquad \forall \; t \,\mathcal{E}\, S_o,$$

$$\text{Rank}(\mathbf{W}_i) = 1, \qquad \forall \; i \,\mathcal{E}\, S_l,$$

$$\sum_{i \in S_l} \sum_{j \in 1}^{n} \; \eta_i \, r_j^H \mathbf{W}_i \, r_j - P_q \leq 0,$$

where, $\mathbf{W}_i = w_i w_i^H$ is a positive semi definite matrix with unit rank. Here r_j is the unit identity vector of order $n \times 1$ which has 1 at the j-th position and 0 everywhere with n as beamforming vector length. Next we discuss, linear matrix inequality based s-procedure to formulate (9) to SDP form.

Lemma 1 (s-procedure [14]): Let $\emptyset_i(e)$, for i=0, 1, be defined as

$$\emptyset_i(e) = e^H \mathbf{A}_i e + b_i^H e + e^H b_i + c_i,$$

where, $\mathbf{A}_i \in \mathbb{H}^{M \times M}$, $b_i \in \mathbb{C}^M$ and $c_i \in \mathbb{R}$. Suppose, there exists an $\hat{e} \in \mathbb{C}^M$ such that $\emptyset_1(\hat{e}) < 0$. Then the following two conditions are equivalent:

1)$\emptyset_0(\mathrm{e}) \geq 0$ and $\emptyset_1(\mathrm{e}) \leq 0$ are satisfied for all e

2) There exists $\lambda \geq 0$ such that

$$\begin{bmatrix} \mathbf{A}_0 & \mathrm{b}_0 \\ \mathrm{b}_0^{\mathrm{H}} & \mathrm{c}_0 \end{bmatrix} + \lambda \begin{bmatrix} \mathbf{A}_1 & \mathrm{b}_1 \\ \mathrm{b}_1^{\mathrm{H}} & \mathrm{c}_1 \end{bmatrix} \geq 0.$$

The first four constraints of problem (9) can be written as

$$\hat{\mathrm{h}}_{\mathrm{i}}^{\mathrm{H}}\mathbf{W}_{\mathrm{i}}\hat{\mathrm{h}}_{\mathrm{i}} + \hat{\mathrm{h}}_{\mathrm{i}}^{\mathrm{H}}\mathbf{W}_{\mathrm{i}}\mathrm{e}_{\mathrm{i}} + \mathrm{e}_{\mathrm{i}}^{\mathrm{H}}\mathbf{W}_{\mathrm{i}}\hat{\mathrm{h}}_{\mathrm{i}} + \mathrm{e}_{\mathrm{i}}^{\mathrm{H}}\mathbf{W}_{\mathrm{i}}\mathrm{e}_{\mathrm{i}} - \mathrm{z} \geq 0, \qquad (3.10)$$

$$\mathrm{e}_{\mathrm{i}}^{\mathrm{H}}\mathrm{Q}_{\mathrm{i}}\mathrm{e}_{\mathrm{i}} - 1 \leq 0, \qquad \forall\ \mathrm{i} \in \mathrm{S}_{l},$$

$$-\sum_{\mathrm{t}\varepsilon\,\mathrm{S_o}} \sum_{\mathrm{i}\varepsilon \mathrm{S}_{l}} [\ \mu_{\mathrm{t}}\ \hat{\mathrm{g}}_{\mathrm{t}}^{\mathrm{H}}\,\mathbf{W}_{\mathbf{i}}\ \hat{\mathrm{g}}_{\mathrm{t}} + \hat{\mathrm{g}}_{\mathrm{t}}^{\mathrm{H}}\,\mathbf{W}_{\mathrm{i}}\mathrm{e}_{\mathrm{t}} + \mathrm{e}_{\mathrm{t}}^{\mathrm{H}}\mathbf{W}_{\mathrm{i}}\hat{\mathrm{g}}_{\mathrm{t}} + \mathrm{e}_{\mathrm{t}}^{\mathrm{H}}\mathbf{W}_{\mathrm{i}}\mathrm{e}_{\mathrm{t}}] + \mathrm{K} \geq 0, \qquad (3.11)$$

$$\mathrm{e}_{\mathrm{t}}^{\mathrm{H}}\mathrm{Q}_{\mathrm{t}}\mathrm{e}_{\mathrm{t}} - 1 \leq 0, \qquad \forall\ \mathrm{t}\ \mathcal{E}\ \mathrm{S_o},$$

As per Lemma1, the pairs of inequalities in (10) and (11) hold if and only if there exist $\lambda_{\mathrm{i}} \geq 0$ and $\lambda_{\mathrm{t}} \geq 0$ such that the matrix inequalities in (12), indicated in the following page hold. From the second inequalities in (10) and (11) one can understand that the condition $\emptyset_1(\hat{\mathrm{e}}) < 0$ is trivially satisfied. Hence the optimization problem in (9) can be equivalently rewritten as

$$\min \quad -\mathrm{z} \qquad (3.13)$$

$$\text{s.t} \qquad \eta_{\mathrm{i}} \geq 0, \qquad \forall\ \mathrm{i}\ \mathcal{E}\ \mathrm{S}_{l},$$

$$\eta_{\mathrm{t}} \geq 0, \qquad \forall\ \mathrm{t}\ \mathcal{E}\ \mathrm{S_o},$$

$$\lambda_{\mathrm{i}} \geq 0, \qquad \forall_{\mathrm{i,t}}$$

$$\lambda_{\mathrm{t}} \geq 0, \qquad \forall_{\mathrm{i,t}}$$

$$\mathrm{Rank}(\mathbf{W}_{\mathrm{i}}) = 1, \qquad \forall\ \mathrm{i}\ \mathcal{E}\ S_{l},$$

$$\sum_{\mathrm{i}\in \mathrm{S}_{l}} \sum_{\mathrm{j}\in 1}^{\mathrm{n}} \eta_{\mathrm{i}}\ \mathrm{r}_{\mathrm{j}}^{\mathrm{H}}\mathbf{W}_{\mathrm{i}}\ \mathrm{r}_{\mathrm{j}} - \mathrm{P_q} \leq 0.\ \forall\ \mathrm{i}\ \mathcal{E}\ \mathrm{S}_{l}$$

Here we can observe that the rank constraint in (13) is not convex. The problem in (13) can be solved by removing the fifth non–convex constraint and solving the remaining convex problem using numerical optimization packages, e.g., CVX [21] solver, and finally keeping only the rank one solutions for $\mathbf{W}_i$.

$$\mathfrak{m}_i = \begin{bmatrix} \mathbf{W}_i + \lambda_i Q_i & \mathbf{W}_i \hat{h}_i \\ \hat{h}_i^H \mathbf{W}_i & \hat{h}_i^H \mathbf{W}_i \hat{h}_i - z - \lambda_i \end{bmatrix} \geq 0, \quad \forall\, i\, \varepsilon\, S_l$$

(3.12)

$$\mathfrak{m}_t = \begin{bmatrix} -\sum_{t \varepsilon S_o} \sum_{i \varepsilon S_l} \mu_t\, \mathbf{W}_i + \lambda_t Q_t & -\sum_{t \varepsilon S_o} \sum_{i \varepsilon S_l} \mu_t\, \mathbf{W}_i\, \hat{g}_t \\ -\sum_{t \varepsilon S_o} \sum_{i \varepsilon S_l} \mu_t\, \hat{g}_t^H\, \mathbf{W}_i & -\sum_{t \varepsilon S_o} \sum_{i \varepsilon S_l} \mu_t\, \hat{g}_t^H\, \mathbf{W}_i \hat{g}_t + K - \lambda_t \end{bmatrix} \geq 0, \quad \forall\, t\, \varepsilon\, S_0$$

Chapter-4

4. SIMULATION RESULTS

4.1. Simulation set up

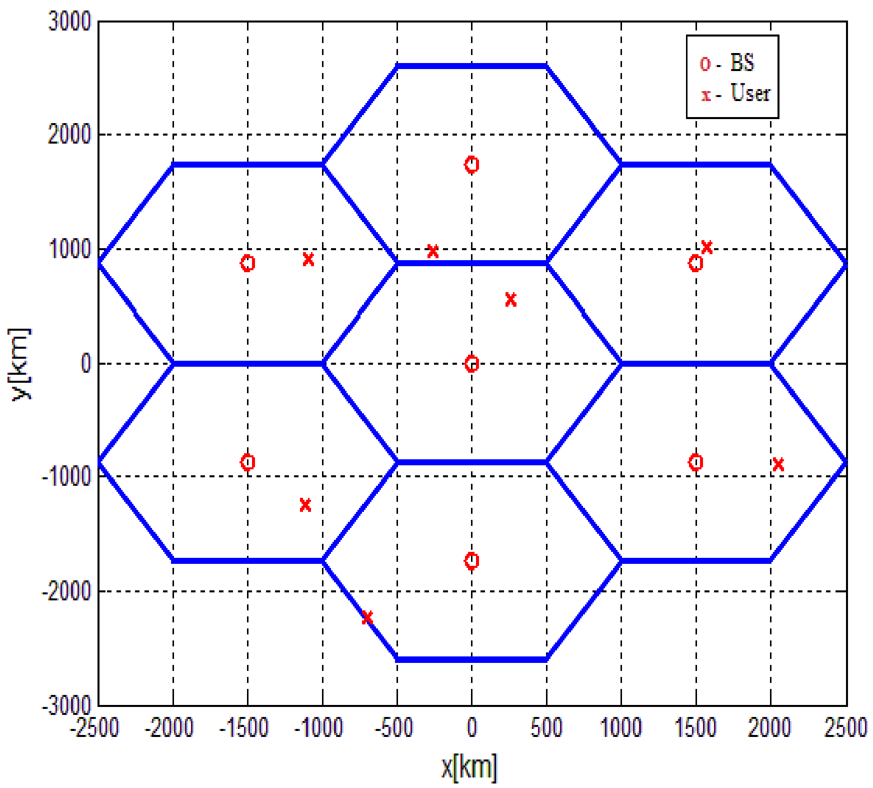
In this section, we evaluate performance of cellular network in terms of improvement in capacity of desired user by using the solution of the problem in (13).In the first step, we generate the 7-cell cluster and users, in every cell of the cluster. Fig.1 shows the example of one user distribution with 7 users. Monte Carlo simulations have been performed over the distribution of one user and we produce 100 uncertain channel realization per user satisfying $\| e_t \|^2 \leq \varepsilon^2$ with different ε values. Here we employ channel model used in [14] which is given as

$$g_t = 10^{-(128.1+37.6\ \log_{10}(l))/20}.\ \Psi_t.\varphi_t.(\hat{g}_{t+}e_t) \qquad (4.1)$$

where, the distance between the BS and the user is denoted by "l", Ψ_t and φ_t are the shadowing and antenna gain respectively.$\hat{g}_t$ and e_t indicate the estimated CSI and CSI error corresponding to i th user. Here we choose a spherical uncertainty set with error radius i.e $Q_b = \varepsilon^{-2} I_M$ for all b = t,i.

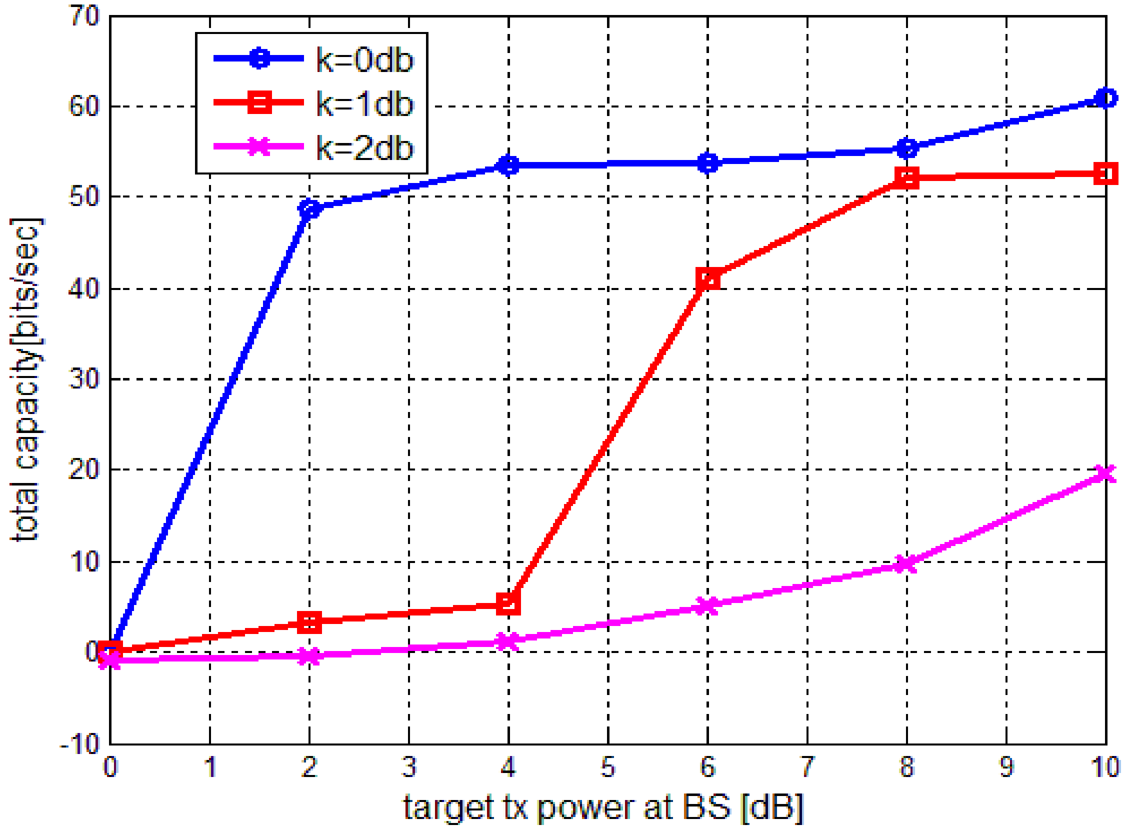
4.2. Performance evaluation

In this sub-section, we examine the performance of robust downlink beamforming for the maximization of capacity of the desired user in the presence of imperfect CSI. Fig.2, Fig.3 and Fig4 illustrates the plot of target power level at BS versus total capacity of the desired user. In Fig.2 we compare the capacity of desired user with different upper bounds for interference



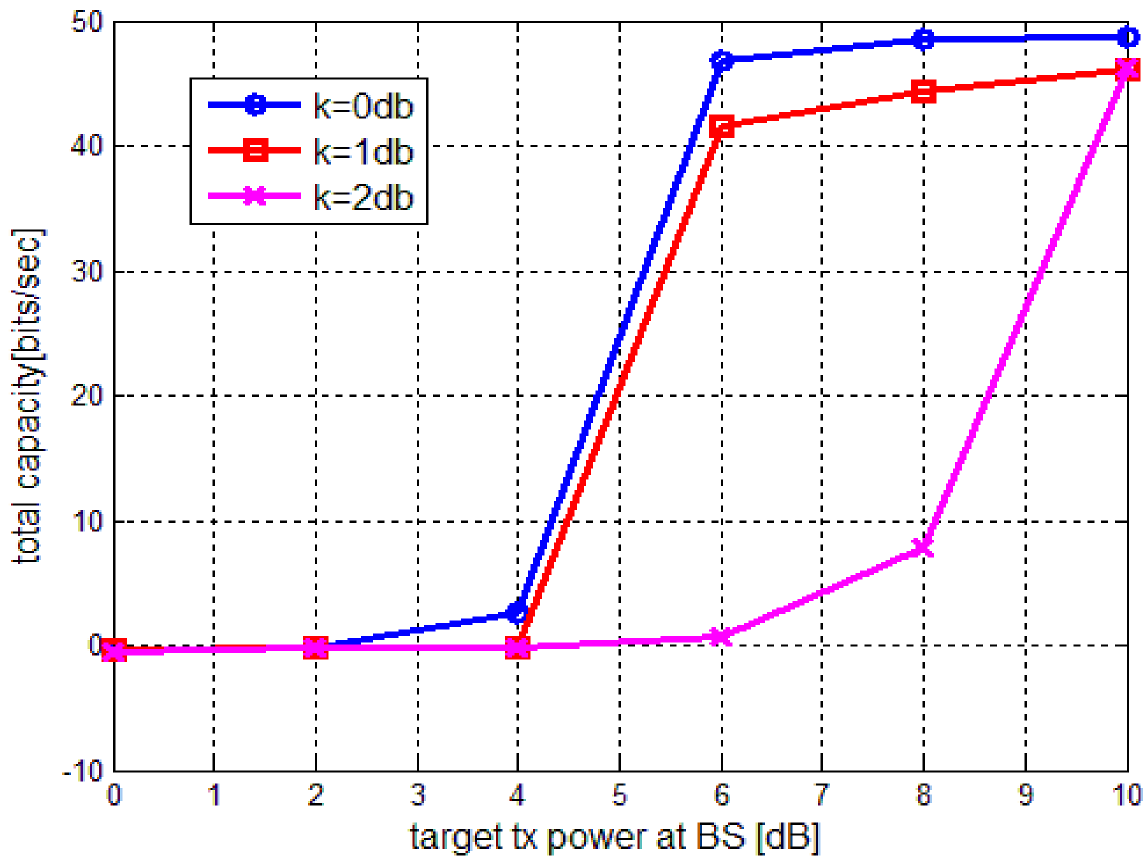
power (K) induced on the other user by varying the total power at BS in seven cell scenario with one user per cell and an error radius ε =0.5.In Fig.3and Fig.4, we perform similar analysis which has been done in Fig.2, but with different error radius ε =0.1 and ε =0.05 respectively. From Fig.2, Fig3 and Fig4, it can be observed as upper bound for interference increases total capacity decreases. Hence the upper minimum as possible. But for particular minimum upper bound for interference power, the capacity will be the capacity will be saturated. Hence from the results it can be observed that solution to the proposed problem gives high capacity and power efficiency for a cellular network.

4.3. Results



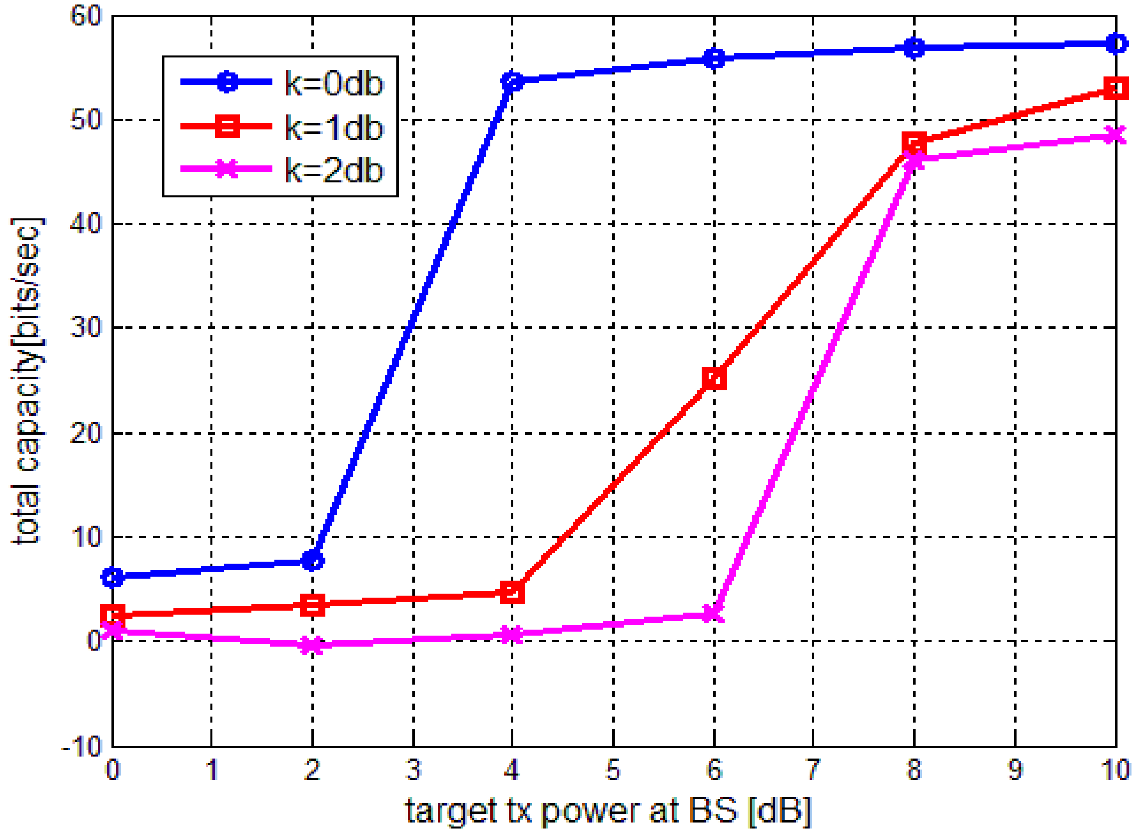
Total capacity versus targeted transmit power values in 7-cell scenario with one user per cell and 4 antenna elements per BS for error radius, ε=0.05

Figure shown above illustrates the plot of target power level at BS versus total capacity of the desired user. Here we compare the capacity of desired user with different upper bounds for interference power (K) induced on the other user by varying the total power at BS in seven cell scenario with one user per cell and an error radius ε =0.05. From above result, it can be observed that, as upper bound for interference increases total capacity decreases. Hence the upper bound should be as minimum as possible. But for particular minimum upper bound for interference power, the capacity will be the capacity will be saturated.



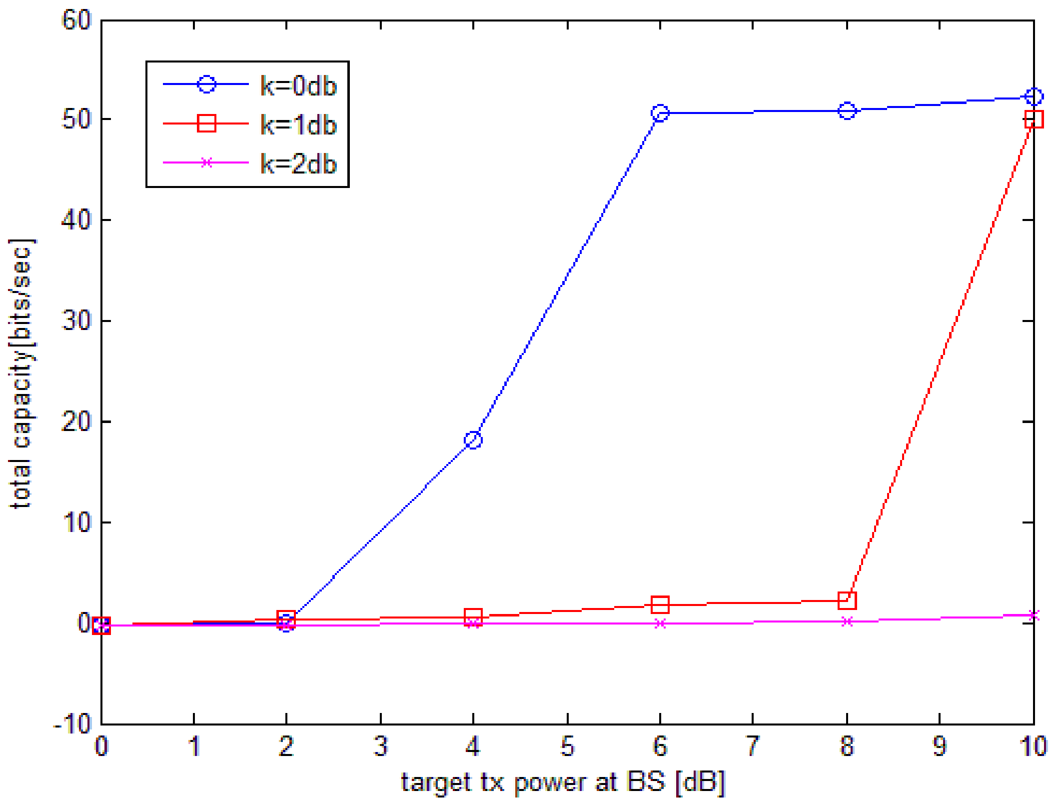
Total capacity versus targeted transmit power values in 7-cell scenario with one user per cell and 4 antenna elements per BS for error radius, ε=0.1

Figure shown above illustrates the plot of target power level at BS versus total capacity of the desired user. Here we compare the capacity of desired user with different upper bounds for interference power (K) induced on the other user by varying the total power at BS in seven cell scenario with one user per cell and an error radius ε =0.1. From above result, it can be observed that, as upper bound for interference increases total capacity decreases. Hence the upper bound should be as minimum as possible. But for particular minimum upper bound for interference power, the capacity will be the capacity will be saturated. Hence from the above result it can be observed that solution to the proposed problem gives high capacity and power efficiency for a cellular network.



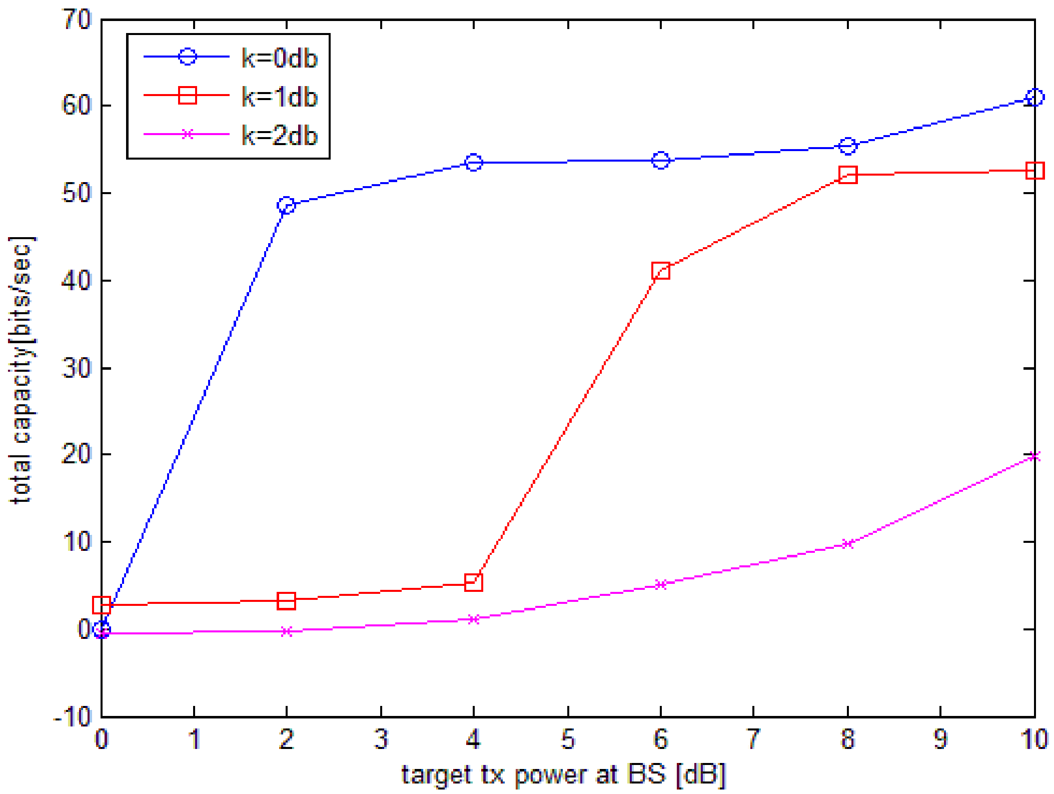
Total capacity versus targeted transmit power values in 7-cell scenario with one user per cell and 4 antenna elements per BS for error radius, ε=0.5

Figure shown above illustrates the plot of target power level at BS versus total capacity of the desired user. Here we compare the capacity of desired user with different upper bounds for interference power (K) induced on the other user by varying the total power at BS in seven cell scenario with one user per cell and an error radius ε =0.5. From above result, it can be observed that, as upper bound for interference increases total capacity decreases. Hence the upper bound should be as minimum as possible. But for particular minimum upper bound for interference power, the capacity will be the capacity will be saturated. Hence from the above result it can be observed that solution to the proposed problem gives high capacity and power efficiency for a cellular network.



Total capacity versus targeted transmit power values in 7-cell scenario with one user per cell and 4 antenna elements per BS for error radius, ε=0.01

Figure shown above illustrates the plot of target power level at BS versus total capacity of the desired user. Here we compare the capacity of desired user with different upper bounds for interference power (K) induced on the other user by varying the total power at BS in seven cell scenario with one user per cell and an error radius ε =0.01. From above result, it can be observed that, as upper bound for interference increases total capacity decreases. Hence the upper bound should be as minimum as possible. But for particular minimum upper bound for interference power, the capacity will be the capacity will be saturated. Hence from the above result it can be observed that solution to the proposed problem gives high capacity and power efficiency for a cellular network.



Total capacity versus targeted transmit power values in 7-cell scenario with one user per cell and 4 antenna elements per BS for error radius, ε=0.005

Figure shown above illustrates the plot of target power level at BS versus total capacity of the desired user. Here we compare the capacity of desired user with different upper bounds for interference power (K) induced on the other user by varying the total power at BS in seven cell scenario with one user per cell and an error radius ε =0.005. From above result, it can be observed that, as upper bound for interference increases total capacity decreases. Hence the upper bound should be as minimum as possible. But for particular minimum upper bound for interference power, the capacity will be the capacity will be saturated. Hence from the above result it can be observed that solution to the proposed problem gives high capacity and power efficiency for a cellular network.

Chapter-5

5. Conclusion and Future work

5.1. Conclusion

This work Maximizes the capacity of the cellular network by increasing the received signal power and reducing the inter-cell interference induced from other cell BSs at every user in the network while satisfying the transmit power constraint in the presence of imperfect CSI. We formulated an optimization problem for improving the capacity of the desired user and later we modify our original non convex problem in to a tractable formulation with convex constraints and LMI. We showed that reformulated problem can be solved easily by using semi definite relaxation. Simulation results have shown that minimum upper bound for interference power gives more capacity for the cellular network and at some minimum upper bound capacity will be saturated. We also observed that higher value for error radius demands more robust system.

5.2. Future Work

Till now we tried for Robust Downlink Beamforming for Capacity Maximization using Semidefinite programming now in future we can do the Robust Downlink Beamforming for Capacity Maximization, using second order conic programming as semidefinite programming has computational complexity and SDP is expensive compared to secondorder conic programming.

References:

[1] Boyd, Stephen, and Lieven Vandenberghe. *Convex optimization*. Cambridge university press, 2004.

[2]Luo, Zhi-Quan, and Wei Yu. "An introduction to convex optimization for communications and signal processing." *Selected Areas in Communications, IEEE Journal on* 24.8 (2006): 1426-1438.

[3] Ben-Tal, Aharon, and Arkadi Nemirovski. "Robust solutions of linear programming problems contaminated with uncertain data." *Mathematical programming* 88.3 (2000): 411-424.

[4] Biegler, Lorenz T. "Solution of dynamic optimization problems by successive quadratic programming and orthogonal collocation." (1983).

[5] Lobo, Miguel Sousa, et al. "Applications of second-order cone programming."*Linear algebra and its applications* 284.1 (1998): 193-228.

[6] Wolkowicz, Henry, Romesh Saigal, and Lieven Vandenberghe, eds. *Handbook of semidefinite programming: theory, algorithms, and applications*. Vol. 27. Springer Science & Business Media, 2000.

[7]Zhang, Qian, and Chenyang Yang. "Transmission mode selection for downlink coordinated multipoint systems." *Vehicular Technology, IEEE Transactions on*62.1 (2013): 465-471.

[8] Brueck, Stefan, et al. "Centralized scheduling for joint transmission coordinated multi-point in LTE-Advanced." *Smart Antennas (WSA), 2010 International ITG Workshop on*. IEEE, 2010.

[9] Papadogiannis, Agisilaos, Eric Hardouin, and David Gesbert. "Decentralising multicell cooperative processing: A novel robust framework." *EURASIP Journal on Wireless Communications and Networking* 2009 (2009): 4.

[10] Gharavol, Ebrahim A., Ying-Chang Liang, and Koen Mouthaan. "Robust downlink beamforming in multiuser MISO cognitive radio networks with imperfect channel-state information." *Vehicular Technology, IEEE Transactions on* 59.6 (2010): 2852-2860.

[11] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson and B. Ottersten, “Convex optimization-based beamforming: From receive to transmit and network designs,” *IEEE Signal Process. Mag,* vol. 27, no. 3, pp. 62-75, 2010.

[12] B. K. Chalise, S. Shahbazpanahi, A. Czylwik and A. B. Gershman, “Robust downlink beamforming based on outage probability specifications,” *Wireless Communications, IEEE Transactions on,* vol. 6, no. 10, pp. 3498-3503, 2007.

[13] S. Han, Q. Zhang and C. Yang, “Distributed coordinated multi-point downlink transmission with over-the-air communication,” in *Communications and Networking in China (CHINACOM), 2010 5th International ICST Conference on*, 2010.

[14] Shen, Chao, et al. "Distributed robust multicell coordinated beamforming with imperfect CSI: an ADMM approach." *Signal Processing, IEEE Transactions on*60.6 (2012): 2988-3003.

[15] Karakayali, M. Kemal, Gerard J. Foschini, and Reinaldo A. Valenzuela. "Network coordination for spectrally efficient communications in cellular systems." *Wireless Communications, IEEE* 13.4 (2006): 56-61.

[16] M. B. Shenouda and T. N. Davidson, “Convex conic formulations of robust downlink precoder designs with quality of service constraints,” *Selected Topics in Signal Processing, IEEE Journal of,* vol. 1, no. 4, pp. 714-724, 2007.

[17] M. Tshangini and M. R. Nakhai, “Robust downlink beamforming with imperfect CSI,” in *Communications (ICC), 2013 IEEE International Conference on*, 2013.

[18] A. Tajer, N. Prasad and X. Wang, “Robust beamforming for multi-cell downlink transmission,” in *Information Theory Proceedings (ISIT), 2010 IEEE International Symposium on*, 2010.

[19] J. Ni, Z. Fei, K.-K. Wong, C. Xing, J. Kuang and K.-F. Tong, “Robust coordinated beamforming for secure MISO interference channels with bounded ellipsoidal uncertainties,” *Wireless Communications Letters, IEEE,* vol. 2, no. 4, pp. 407-410, 2013.

[20] Q. Li, A. M.-C. So and W.-K. Ma, “Distributionally robust chance-constrained transmit beamforming for multiuser MISO downlink,” in *Acoustics, Speech and Signal Processing (ICASSP), 2014 IEEE International Conference on*, 2014.

[21] Boyd, Stephen, and Lieven Vandenberghe. *Convex optimization*. Cambridge university press, 2004.